NASA TM-85984

NASA Technical Memorandum 85984

Numerical Simulation of Boundary-Layer Transition

P.R. Spalart

June 1984

National Aeronautics and
Space Administration

NASA Technical Memorandum 85984

Numerical Simulation of Boundary-Layer Transition

P R Spalart, Ames Research Center, Moffett Field, California

National Aeronautics and
Space Administration

Ames Research Center
Moffett Field California 94035

N84-28746 #

NUMERICAL SIMULATION OF BOUNDARY-LAYER TRANSITION

P. R. Spalart
NASA Ames Research Center
Moffett Field, CA 94035 USA

I. FORMULATION

The transition to turbulence in boundary layers is investigated by direct numerical solution of the nonlinear, three-dimensional, incompressible Navier-Stokes equations in the half-infinite domain over a flat plate. Periodicity is imposed in the x-direction (streamwise) and in the z-direction (spanwise); the y-coordinate extends from 0 to ∞.

The simulation is periodic in the x-direction, unlike the experiments in which the boundary-layer thickness grows in the x-direction. A forcing term is added to the x-momentum equation that approximates the convection terms associated with this spatial growth. The approximation is based on boundary-layer assumptions (applied only to the mean flow) and the self-similarity of the mean-velocity profile. With this forcing applied, the laminar velocity profile, instead of becoming an error function and thickening without bounds, is a Blasius profile. Thus, the stability characteristics are very close to the experimental characteristics. Furthermore, the equation can be written in a moving reference frame, so that the boundary-layer thickens in time while retaining a Blasius profile. The procedure of adding a forcing term allows the disturbances to extract energy from the mean flow, and is much preferable to a procedure in which the mean-velocity profile would be imposed.

The spatial representation is spectral in all directions [1]. The basis functions that are used represent divergence-free velocity fields and satisfy the boundary conditions as suggested by Leonard and Wray [2]. Leonard and Wray applied a weak formulation, which eliminates the pressure and allows an accurate and straightforward time-advance scheme. Leray's weak formulation is used here [3]. An advantage of this formulation over Leonard's is that it keeps the numerical Stokes operator real, symmetric, and negative-definite. On the other hand, Chebyshev polynomials cannot be used.

The x- and z-directions are treated by Fourier series. In the y-direction, the velocity field is first split into "irrotational" and "vortical" components in order to better accommodate two different length scales. The length scale of the vortical component is δ, the thickness of the boundary layer. The length scale of the irrotational components is Λ, the wavelength in the (x,z) plane, which is significantly larger than δ. This irrotational component can be represented by a single exponential function for each horizontal wave-vector. To represent the vortical component, an exponential mapping is applied from $[0,\infty[$ into $[0,1[$, and shifted Jacobi polynomials are used in the transformed coordinate. The vortical component is infinitely differentiable over the closed interval $[0,1]$, so that the convergence of the polynomial method will be faster than algebraic. The cost of the transforms from

real space to Jacobi space is of the order of N^2. Figure 1 is a plot of the first few basis-functions versus y. All the functions decay exponentially as $y \to \infty$ but the first function, which includes the irrotational component, decays much more slowly than the other ones.

The time-advance scheme is hybrid and second-order accurate. The convection terms are treated by a Runge-Kutta scheme which is explicit, third-order accurate, and conditionally stable; the Stokes terms are treated by the Crank-Nicolson scheme.

II. RESULTS

In order to check the convergence of the method, the Orr-Sommerfeld equation was solved for a Blasius profile and for a real wave-number. This problem is known to produce a few discrete eigenvalues and a continuous spectrum on the $C_r = 1$, $C_i < 0$ axis [4]. Figure 2 is a contour plot of the error in the principal discrete eigenvalue as a function of Yo, the length scale of mapping, and Ny, the number of points in the y-direction. The convergence as $Ny \to \infty$ with Yo fixed is very fast. It is expected to be faster than algebraic, but not as fast as exponential [5]. The plot also indicates the optimum value of Yo: about $2\delta^*$. Figure 3 shows that the numerical spectrum includes a string of eigenvalues that becomes denser and tends to the $C_r = 1$, $C_i < 0$ axis. Its convergence is much slower than that for the discrete eigenvalues; the reason is that the corresponding eigenfunctions behave like sine waves as $y \to \infty$, which makes them hard to approximate with the expansion functions in Fig. 1.

The early nonlinear stages of transition of a Blasius boundary layer, disturbed by a vibrating ribbon, were then simulated in three dimensions. A two-dimensional Tollmein-Schlichting (TS) wave of finite amplitude was introduced in the initial field, as well as three-dimensional white noise of much lower energy. The streamwise period was twice that of the TS wave; the spanwise period was chosen much longer to avoid constraining the spectrum. Spanwise lines of particles were introduced, near the critical layer, to simulate the smoke lines used in experiments.

Figure 4 summarizes the time-evolution of the flow. The energy of the fundamental TS wave and the energy carried by all the other wave-vectors are plotted separately. The TS wave grows from branch I to branch II of the TS stability diagram, then starts decaying. The energy of the other modes remains small until after the flow crosses branch II; then it grows very rapidly, and nonlinear interactions take place. The shape factor H of the boundary layer remains at the Blasius value of 2.6 until transition occurs; then it rapidly decreases. The agreement with Kachanov's experiments is excellent [6].

Simulations were conducted with the same background noise, but different values for the TS wave amplitude. Figure 5 contains top views of the particles in the boundary layer. If the maximum TS wave amplitude is less than 0.3%, transition does not occur. In Fig. 5(a), with an amplitude of 0.9%, three-dimensional breakdown occurs and is of the subharmonic or "H" type (the lambda-shaped particle lines are staggered).

In Fig. 5(b), with amplitude 5%, the lambda patterns are not staggered, indicating a Klebanoff-type breakdown. The patterns appear "broken," a result of the randomness of the initial three-dimensional disturbance. The qualitative agreement with Saric's experiments is good [7].

Figure 6 is a plot of the spectrum in an (x,z) plane at the beginning of an H-type breakdown. The fundamental TS wave still dominates the spectrum; its higher harmonic is also present. The growing three-dimensional subharmonic component is obvious; the wave number and the broadband character of the instability agree very well with Herbert's small disturbance theory [8].

III. REFERENCES

1. Gottlieb, D. and Orszag, S. A., "Numerical Analysis of Spectral Methods," NSF-CMBS Monograph No. 26, Society of Industrial and Applied Mathematics, Philadelphia, Penn., 1977.
2. Leonard A. and Wray, A., "A New Numerical Method for the Simulation of Three-Dimensional Flow in a Pipe," NASA TM-84267, 1982.
3. Temam, R., "Navier-Stokes Equations and Nonlinear Functional Analysis," NSF-CMBS Monograph No. 41, Society of Industrial and Applied Mathematics, Philadelphia, Penn., 1983.
4. Grosch, C. E. and Salwen, H., "The Continuous Spectrum of the Orr-Sommerfeld Equation. Pt. 1. The Spectrum and the Eigenfunctions," J. Fluid Mech., Vol. 87, Pt. 1, 1978, pp. 33-54.
5. Boyd, J. P., "The Optimization of Convergence for Chebyshev Polynomial Methods in an Unbounded Domain," J. Comp. Phys., Vol. 45, No. 1, 1982, pp. 43-79.
6. Kachanov, Y. S. and Levchenko, V. Y., "The Resonant Interaction of Disturbances at Laminar-Turbulent Transition in a Boundary Layer," J. Fluid Mech., Vol. 138, 1984, pp. 209-247.
7. Saric, W. S., Kozlov, V. V., and Levchenko, V. Y., "Forced and Unforced Subharmonic Resonance in Boundary-Layer Transition," AIAA Paper 84-0007, 1984.
8. Herbert, T., "Analysis of the Subharmonic Route to Transition in Boundary Layers," AIAA Paper 84-0009, 1984.

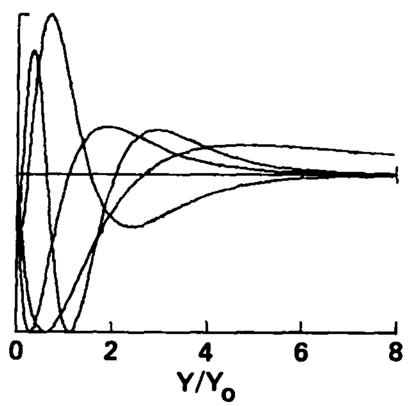


Fig. 1 First four basis-functions.

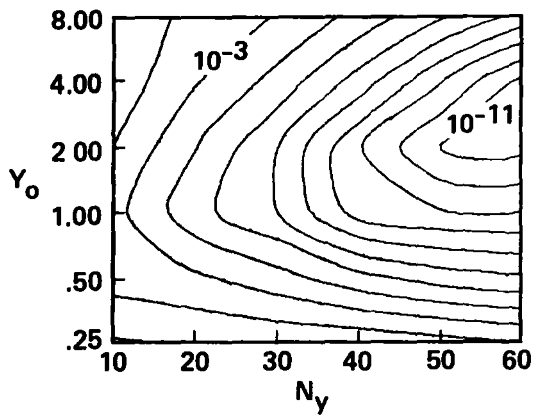
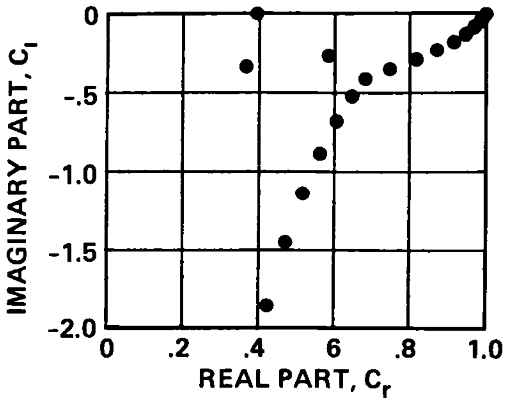
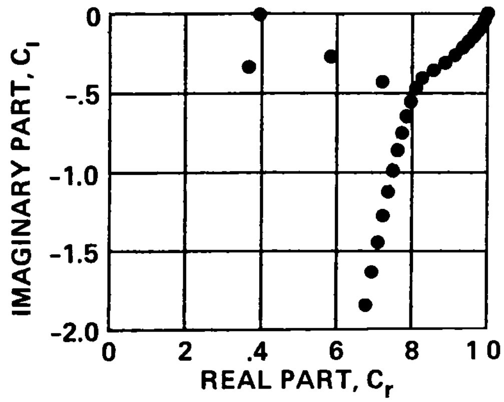


Fig. 2 Error in Orr-Sommerfeld eigenvalue.



(a) Ny = 50.



(b) Ny = 100.

Fig. 3 Numerical spectrum: Yo = 1.5.

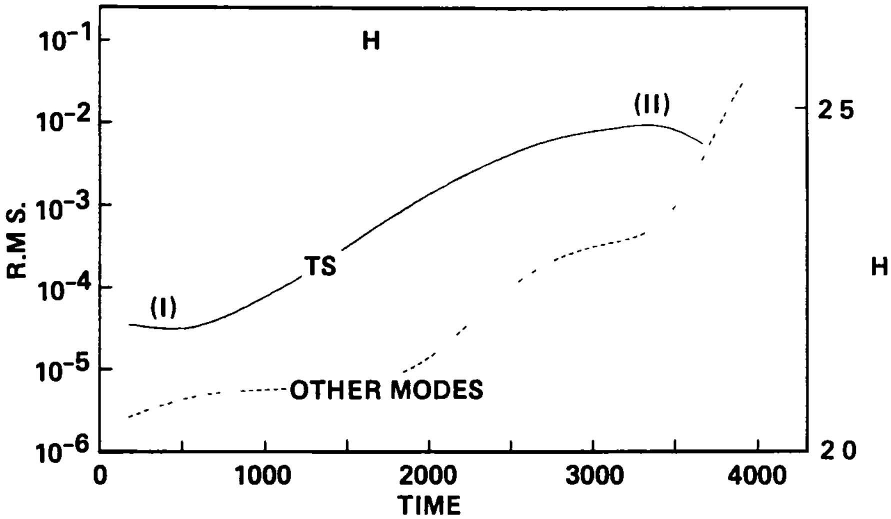
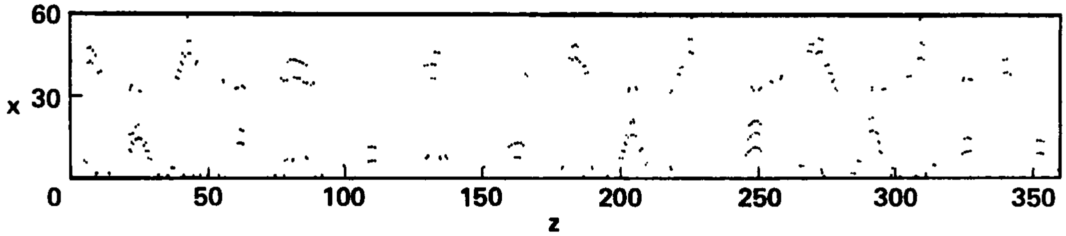
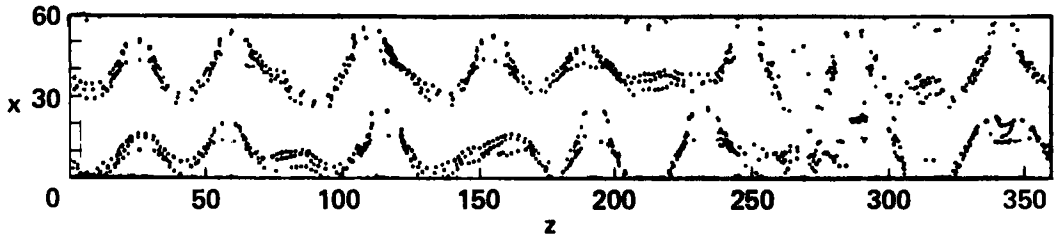


Fig. 4 Time-evolution of the disturbance energy and of the shape factor.



(a) Amplitude = 0.9%.



(b) Amplitude = 5%.

Fig. 5 Smoke lines in transitioning boundary layer.

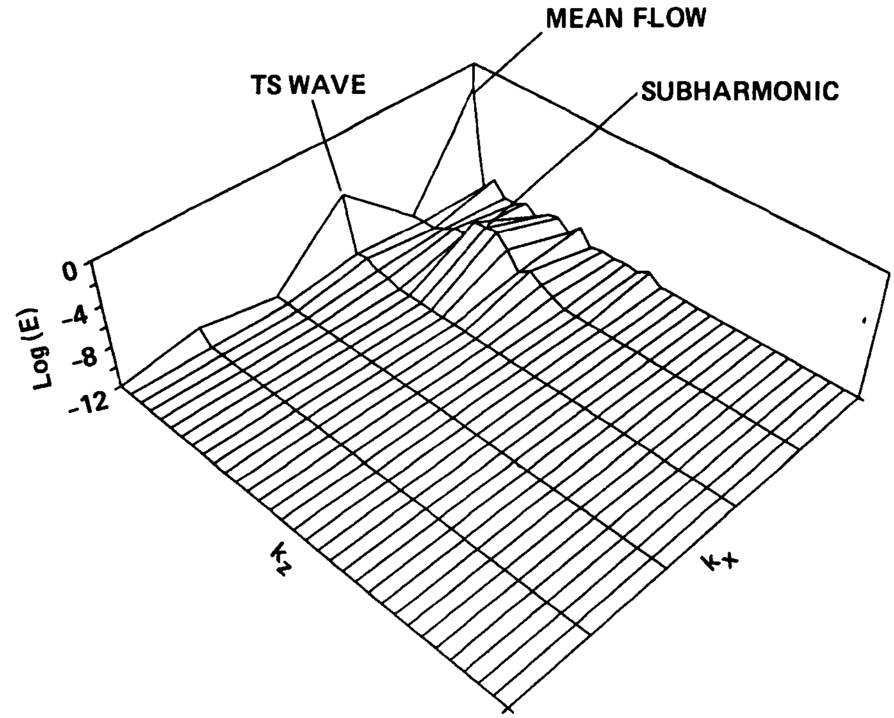


Fig. 6 Two-dimensional spectrum.

1 Report No	2 Government Accession No	3 Recipient's Catalog No
NASA TM-85984		

4 Title and Subtitle	5 Report Date
NUMERICAL SIMULATION OF BOUNDARY-LAYER TRANSITION	June 1984
	6 Performing Organization Code
7 Author(s)	8 Performing Organization Report No
P. R. Spalart	A-9796
9 Performing Organization Name and Address	10 Work Unit No
Ames Research Center Moffett Field, CA 94035	T-6465
	11 Contract or Grant No
12 Sponsoring Agency Name and Address	13 Type of Report and Period Covered
National Aeronautics and Space Administration Washington, DC 20546	Technical Memorandum
	14 Sponsoring Agency Code
	505-31-01

15 Supplementary Notes

Point of Contact: P. R. Spalart, Ames Research Center, MS 202A-1, Moffett Field, CA 94035 (415) 965-6667 or FTS 448-6667.

16 Abstract

The transition to turbulence in boundary layers was investigated by direct numerical solution of the nonlinear, three-dimensional, incompressible Navier-Stokes equations in the half-infinite domain over a flat plate. Periodicity was imposed in the streamwise and spanwise directions. A body force was applied to approximate the effect of a nonparallel mean flow. The numerical method was spectral, based on Fourier series and Jacobi polynomials, and used divergence-free basis functions. Extremely rapid convergence was obtained when solving the linear Orr-Sommerfeld equation. The early nonlinear and three-dimensional stages of transition, in a boundary layer disturbed by a vibrating ribbon, were successfully simulated. Excellent qualitative agreement was observed with either experiments or weakly nonlinear theories. In particular, the breakdown pattern was staggered or nonstaggered depending on the disturbance amplitude.

17 Key Words (Suggested by Author(s))	18 Distribution Statement
Boundary layer Transition Numerical method	Unlimited Subject Category - 02

19 Security Classif (of this report)	20 Security Classif (of this page)	21 No of Pages	22 Price*
Unclassified	Unclassified	7	A02

*For sale by the National Technical Information Service, Springfield, Virginia 22161

End of Document